

Section 3.14 Problems, Chapter 3 173

- (3) The Browns wish to accumulate at least \$150,000 at the time of their last deposit in a college fund for their daughter by contributing an amount A into the account at the end of each year for eighteen years. What is the smallest annual payment A that will suffice if the college fund earns a level annual effective interest rate of 5%? If at the end of ten years, it is announced that the annual effective interest rate will drop to 4.5%, how much must the Browns increase their payments in order to reach their accumulation goal? Assume that the Browns wish to continue to make level payments except for a slightly reduced final payment.
- (4) Elwood wishes to purchase a home. She has saved up \$13,200 for a down payment. Based on her earnings, she qualifies for a thirty-year mortgage with level monthly payments of \$820 including escrow and a nominal interest rate convertible monthly of 5.85%. Her payments are due at the end of each month. From each payment, \$240 will be put aside in an escrow account for the payment of taxes and homeowners insurance. What is the most expensive house Elwood can buy if her aunt has promised to give her the money needed for loan applications, inspections, and all other required buyer's closing costs?
- (5) Matt inherited as a trust a fifteen-year annuity-immediate with annual payments. He has been told that the annuity payments earn compound interest at a level rate and that at the end of fifteen years, their accumulated value will be \$37,804.39. He has further been assured that figured at this same rate of interest, the value of his inheritance was \$15,077.10. The trust executor will not reveal the amount of the annual payments. Determine this amount and also the annual effective interest rate earned by the annuity payments.
- (6) Mrs. Williams finds that she has two options for investing \$32,000.02 for fifteen years. The first option is to deposit the \$32,000.02 into a fund earning a nominal rate of discount $d^{(4)}$ payable quarterly. The second option is to purchase an annuity-immediate with 15 level annual payments, the annuity payments computed using an annual effective rate of 7%, and then when she gets an annuity payment, to immediately invest it into a fund earning an annual effective rate of 5%. Mrs. Williams calculates that the second option produces an accumulated value that is \$1,500 more than the accumulated value yielded by the first option. Calculate $d^{(4)}$.
- (7) A buyer of a 2003 Protege S Hatchback has a choice of 0% financing for 60 months or a \$3,600 rebate. He plans to make no down payment. The buyer is able to qualify for 7% annual effective financing through his credit union and thereby take advantage of the rebate. Let Y denote his negotiated price for the Protege S Hatchback. How large must Y be in order for the 0% dealer financing to be preferable?

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- (8) Sigmund and Karl each borrowed an identical amount from Ludwig at a nominal rate of discount of 5.4% convertible quarterly. Sigmund repays his loan by making payments of \$2,000 at the end of each year for six years. Karl makes payments of \$3,200 at four equally spaced times T , $2T$, $3T$, and $4T$. Find T . [HINT: You will need to find the interest rate i for a period of length T .]

(3.3) Annuities-due

- (1) April received an inheritance from her grandmother in the form of an annuity. The annuity pays \$3,000 on January 1st from 1966 through 1984. Find the value of this annuity on January 1, 1966 using an annual effective interest rate of 5% and represent this value by an appropriate annuity symbol.
- (2) Suppose $i = 3\%$. Find the value one month before the first payment of a level annuity-due paying \$200 at the beginning of each month for five years.
- (3) (a) Describe in words what the difference $a_{\overline{n+1}|i} - a_{\overline{n}|i}$ is measuring.
(b) Given that $a_{\overline{n+1}|i} - a_{\overline{n}|i} = .177208656$ and $\ddot{a}_{\overline{n+1}|i} - \ddot{a}_{\overline{n}|i} = .185248436$, find the integer n .
- (4) Steven Wong wishes to save for his retirement by depositing \$1,200 at the beginning of each year for thirty years. Exactly one year after his last deposit, he wishes to begin making annual level withdrawals until he has made twenty withdrawals and exhausted the savings. Find the amount of each withdrawal if the effective interest rate is 5% during the first thirty years but only 4% after that.
- (5) Starting on his 25th birthday and continuing through his 60th birthday, Fred deposits \$7,500 each year on his birthday into a retirement fund earning an annual effective rate of 5%. Immediately after the last deposit, the accumulated value of the fund is transferred to a fund earning an annual effective rate of j . Five years later, a twenty-five year annuity-due paying \$5,800 each month is purchased with the funds. The purchase price of the annuity was determined using an annual effective rate of interest of 4%. Find j .
- (6) Given $\ddot{a}_{\overline{n}|i} = 12$ and $\ddot{a}_{\overline{2n}|i} = 21$, find $a_{\overline{4n}|i}$.
- (7) Given $\ddot{a}_{\overline{n}|i} = 31.61667882$ and $s_{\overline{n+1}|i} = 64,024.90944$, find i and n .
- (8) Given $a_{\overline{n-1}|i} = 11.38229339$ and $s_{\overline{n+1}|i} = 414.3137296$, find i and n .

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of payment at time t is $(2 + t)^2$ and the force of interest is $(1 + t)^{-1}$. Find the present value of this annuity.

- (4) Stacey and her husband David have a joint savings account that earns 3.5% interest payable continuously and has a current balance of \$58,458. Each year, David wishes to withdraw \$4,000 payable continuously at a level rate. Stacey wishes to deposit \$ X at the beginning of each year (for thirty years) so that the account will last for thirty years. What is the least \$ X that will work?
- (5) A continuously paying level annuity pays \$72 each year for twenty years. The force of interest at time t is $\frac{5}{3+2t}$. Find the present value of this annuity.
- (6) Quang deposits \$20,000 in a savings account with a discount rate of 4.8% convertible quarterly. He leaves his money in this account to accumulate for twelve years, then moves it to a fund which is accumulating at 5.4% per annum convertible continuously. If, starting at time 12 when he invests in the new fund, money is withdrawn levelly and continuously at a rate of \$8,000 per annum, how long will Quang’s money last?
- (7) This problem concerns the annuity underlying the annuity symbols $(I^{(m)}a)_{\overline{n}|i}^{(m)}$.
 - (a) The rate of payment over an interval is the amount paid normalized by dividing by the length of the interval. Show that this annuity pays at a rate of $\frac{j}{m}$ over the interval $[\frac{j-1}{m}, \frac{j}{m}]$.
 - (b) Show that the payments of this annuity total $\frac{n^2}{2} + \frac{n}{2m}$.
 - (c) Show that the limit $\lim_{m \rightarrow \infty} \frac{n^2}{2} + \frac{n}{2m}$ of the total payment amounts found in (b) is equal to $\int_0^n t dt$, the total payments of the continuously paying annuity underlying the symbol $(\overline{Ia})_{\overline{n}|}$.

(4.7) A yield rate example

- (1) Introduce a function $f(x)$ whose root is the yield rate desired in Example (4.7.1). Apply Newton’s method with initial approximation $x_1 = .052$ to find x_2 and x_3 . Discuss your results.
- (2) Bob deposits \$1,500 at the beginning of each quarter for sixteen years in a fund earning a nominal rate of interest of 6% convertible monthly. The interest from this fund is paid out monthly and can only be reinvested at an effective annual rate of 5.2%. This is just as in Problem (4.5.5). Find Bob’s yield rate for the twenty-year period.

